THE ANALYSER

The "Clarke Test Number" which gives, in effect, a measure of the "goodness" of a Bombe story, is worked out as follows.

Suppose we are running a menu of L links (all in one chain, with no auxiliary or subsidiary chains), and that a given story on the menu has p self-couples and r different couples (a confirmation, of course, counting only once).

Let N be the total number of possible 10-couple keys ($N=1{\cdot}507.10^{14}$). Let M(p,r) be the number of these 10-couple keys which contain the p self-couples and the r couples of the particular story we are considering.

M(p,r) is in fact equal to,

$$\frac{(26-\overline{p+2r})!}{(6-p)!\;2^{10-r}(10-r)!}$$

The chance of the given story being the correct one is proportional to,

$$\frac{M(p,r).25^{L}}{N.26^{3}} = X, \text{say}$$

The Clarke Test Number (C) is not the number X itself but,

C = 5 log X (the logarithm being to base 10)

and this, of course, serves equally well as a test, though the actual values will be different.

Thus if the Clarke Test Number is +14 we have,

$$C = +14$$

$$X = \frac{M(p,r).25^{L}}{N.26^{3}} = 10^{c/5} = 10^{2.8} = 631$$

Thus the chance of the story being right is proportional to 631. What the <u>actual</u> chance is will depend on other factors – e.g. number of turnovers risked, number of wheel-orders to be run, a priori chance of crib being right, etc.

We have, then

$$C = 5\log\left[\frac{M(p,r).25^{L}}{N.26^{3}}\right]$$

$$= 5\left\{\log\frac{M(p,r)}{N,26^{3}} + L\log 25\right\}$$

$$= 5\log\frac{M(p,r)}{N.26^{3}} + 7L \text{ approx}$$

Thus every link on the menu should add 7 units to the Clarke Test Number C, and a table is given below of the values of

$$5\log\frac{M(p,r)}{N.26^3}$$

for varying values of p and r.

N.B. for p more than 6, or r more than 10 the story is impossible, so the machine shows “Clarke Test Number Excessive”.

Table of the values of

$$5\log\frac{M(p,r)}{N.26^3}$$

for different values of p and r.

Values of r		1	2	3	4	5	6	7	8	9	10
	0	-29	-36	-44	-51	-58	-65	-71	-78	-85	-92
	1	-32	-39	-46	-53	-60	-66	-73	-79	-86	-92
Values	2	-35	-42	-49	-56	-62	-69	-75	-81	-86	-92
of	3	-39	-46	-52	-59	-65	-71	-77	-82	-87	-92
p	4	-43	-50	-56	-62	-68	-74	-79	-84	-88	-92
	5	-48	-54	-61	-66	-72	-77	-82	-86	-90	-92
	6	-54	-61	-66	-72	-77	-82	-86	-90	-92	-92

Examples

(i) Menu with 12 links, story with 2 self-couples and 7 different couples –

$$\begin{aligned} C &= M(2,7) + 7.12 \\ &= -75 + 84 \\ &= +9 \end{aligned}$$

(ii) Menu with 10 links, story with 1 self-couple and 9 different couples –

$$\begin{aligned} C &= M(1,9) + 7.10 \\ &= -86 + 70 \\ &= -16 \end{aligned}$$

O.H.L.

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TOP SECRET

	1	2	3	4	5	6	7	8	9	10
log M(p,r) 0 log $N.26^3$	12.6665 18.4230 6.2435 5 29.2175 – 29	11.1790 18.4230 8.7560 5 37.7800 – 36	9.7193 18.4230 9.2963 5 44.4815 – 44	8.2856 18.4230 11.8626 5 51.3130 – 51	6.8791 18.4230 12.4561 5 58.2805 – 58	5.4983 18.4230 13.0753 5 65.3765 – 65	4.1430 18.4230 15.7200 5 72.6000 – 71	2.7993 18.4230 16.3763 5 79.8815 – 78	1.4472 18.4230 17.0242 5 85.1110 - 85	.0000 18.4230 19.5770 5 93.8850 – 92
log M(p,r) 1 log $N.26^3$	12.0645 18.4230 7.6415 5 32.2075 – 32	10.6149 18.4230 8.1919 5 40.9595 - 39	9.1959 18.4230 10.7729 5 47.8645 – 46	7.8082 18.4230 11.3852 5 54.9260 – 53	6.4533 18.4230 12.0303 5 60.1515 – 60	5.1303 18.4230 14.7073 5 67.5365 – 66	3.8407 18.4230 15.4177 5 73.0885 – 73	2.5775 18.4230 16.1545 5 80.7725 – 79	1.3222 18.4230 18.8992 5 86.4960 - 86	.0000 18.4230 19.5770 5 93.8850 – 92
log M(p,r) 2 log $N.26^3$	11.4014 18.4230 8.9784 5 36.8920 - 35	9.9921 18.4230 9.5691 5 43.8455 – 42	8.6170 18.4230 10.1940 5 50.9700 - 49	7.2765 18.4230 12.8535 5 56.2675 – 56	5.9759 18.4230 13.5529 5 63.7645 – 62	4.7160 18.4230 14.2930 5 69.4650 – 69	3.4983 18.4230 15.0753 5 75.3765 – 75	2.3222 18.4230 17.8992 5 81.4960 – 81	1.1761 18.4230 18.7531 5 87.7655 – 86	.0000 18.4230 19.5770 5 93.8850 – 92
log M(p,r) 3 log $N.26^3$	10.6609 18.4230 8.2379 5 39.1895 – 39	9.2923 18.4230 10.8693 5 46.3465 – 46	7.9633 18.4230 11.5403 5 53.7015 - 52	6.6749 18.4230 12.2519 5 59.2595 – 59	5.4314 18.4230 13.0084 5 65.0420 – 65	4.2380 18.4230 15.8150 5 71.0750 – 71	3.1004 18.4230 16.6774 5 77.3870 – 77	2.0212 18.4230 17.5982 5 83.9910 – 82	1.0000 18.4230 18.5770 5 88.8850 – 87	.0000 18.4230 19.5770 – 92

$$LogN = 14.1781$$
$$\frac{4.2449}{18.4230}$$

TOP SECRET

		1	2	3	4	5	6	7	8	9	10
log m(pr) log $N.26^3$	4	9.8162 18.4230 9.3932 5 44.2175 – 43	8.4914 18.4230 10.0684 5 50.3420 – 50	7.2095 18.4230 12.7865 5 57.9325 - 56	5.9759 18.4230 13.5529 5 63.7645 – 62	4.7952 18.4230 14.3722 5 69.8610 – 68	3.6749 18.4230 15.2519 5 74.2595 – 74	2.6232 18.4230 16.2002 5 79.0010 - 79	1.6532 18.4230 17.2302 5 84.1510 – 84	.7782 18.4230 18.3552 5 89.7760 – 88	.0000 -92
log M(p,v) log $N.26^3$	5	8.8162 18.4230 10.3922 5 49.9660 – 48	7.5378 18.4230 11.1148 5 55.5740 – 54	6.3075 18.4230 13.8845 5 61.4225 – 61	5.1303 18.4230 14.7073 5 67.5365 - 66	4.0170 18.4230 15.5940 5 73.9700 – 72	2.9754 18.4230 16.5524 5 78.7620 - 77	2.0212 -82	1.1761 -86	.4771 18.4230 18.0541 5 90.2705 - 90	0000 -92
log M(p,r) log $N.26^3$	6	7.5378 -54	6.3075 -61	5.1303 -66	4.0170 -72	2.9754 -77	2.0212 -82	1.1761 -86	.4771 -92	.0000 -92	0000 -92

Log $N.26^3$ = 18.4230